5.5 Shannon の第 1 定理の一般化

p. 123 式 5-23 から 5-24

式 5-23 左辺の

$$\sum_{B^n} P(\beta)H(A^n/\beta)$$

は

$$\sum_{\beta\in B^n} P(\beta)H(A^n/\beta)$$

ここで $\beta = (b_{j_1}, b_{j_2}, \ldots, b_{j_n})$ とすると

$$\sum_{(b_{j_1}, b_{j_2}, \ldots, b_{j_n})\in B^n} P(b_{j_1}, b_{j_2}, \ldots, b_{j_n})H(A^n/(b_{j_1}, b_{j_2}, \ldots, b_{j_n}))$$

$$
\begin{aligned}
& H(A^n/\beta) && (1)\\
&= H(A^n/(b_{j_1}, b_{j_2}, \ldots, b_{j_n})) && (2)\\
& n \text{ 個のシンボルは独立だから} \\
&= H(A/b_{j_1}) + H(A/b_{j_2}) + \cdots + H(A/b_{j_n}) && (3)
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{B^n} P(\beta)H(A^n/\beta) && (4)\\
= & \sum_{(b_{j_1}, b_{j_2}, \ldots, b_{j_n})\in B^n} P(b_{j_1}, b_{j_2}, \ldots, b_{j_n})(H(A/b_{j_1}) + H(A/b_{j_2}) + \cdots + H(A/b_{j_n})) && (5)\\
& && (6)
\end{aligned}
$$

の 最初の項の和を考える.

$$
\begin{aligned}
& \sum_{(b_{j_1}, b_{j_2}, \ldots, b_{j_n})\in B^n} P(b_{j_1}, b_{j_2}, \ldots, b_{j_n})H(A/b_{j_1}) && (7)\\
&= \sum_{b_{j_1}\in B}\sum_{b_{j_2}\in B}\cdots\sum_{b_{j_n}\in B} P(b_{j_1})P(b_{j_2})\cdots P(b_{j_n})H(A/b_{j_1}) && (8)\\
&= \sum_{b_{j_1}\in B} P(b_{j_1})H(A/b_{j_1}) \sum_{b_{j_2}\in B}\sum_{b_{j_3}\in B}\cdots\sum_{b_{j_n}\in B} P(b_{j_2})P(b_{j_3})\cdots P(b_{j_n}) && (9)\\
&= \sum_{b_{j_1}\in B} P(b_{j_1})H(A/b_{j_1}) && (10)
\end{aligned}
$$

ゆえに

$$\sum_{B^n} P(\beta)H(A^n/\beta) = n\sum_{b\in B} P(b)H(A/b)$$