2.4　無記憶情報源の拡大

式 2-14

$$H(S^n) = \sum_{\sigma_i \in S^n} P(\sigma_i) \log_2 \frac{1}{P(\sigma_i)} \tag{1}$$

式 2-15

$$\begin{aligned}
\sum_{\sigma_i \in S^n} P(\sigma_i) &= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) && (2)\\
&= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}) P(s_{i_2}) \cdots P(s_{i_n}) && (3)\\
&= \sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q} P(s_{i_1}) P(s_{i_2}) \cdots P(s_{i_n}) && (4)\\
&= \sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_{n-1}=1}^{q} P(s_{i_1}) P(s_{i_2}) \cdots P(s_{i_{n-1}}) \sum_{i_n=1}^{q} P(s_{i_n}) && (5)\\
&= \sum_{i_1=1}^{q} P(s_{i_1}) \sum_{i_2=1}^{q} P(s_{i_2}) \cdots \sum_{i_n=1}^{q} P(s_{i_n}) && (6)\\
&= 1 && (7)
\end{aligned}$$

式 2-16

$$\begin{aligned}
H(S^n) &= \sum_{\sigma_i \in S^n} P(\sigma_i) \log_2 \frac{1}{P(\sigma_i)} \qquad \text{式 (2-14)} && (8)\\
&= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \log_2 \frac{1}{P(s_{i_1}) P(s_{i_2}) \cdots P(s_{i_n})} && (9)\\
&= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \left(\log_2 \frac{1}{P(s_{i_1})} + \log_2 \frac{1}{P(s_{i_2})} + \cdots + \log_2 \frac{1}{P(s_{i_n})} \right)\\
&= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \log_2 \frac{1}{P(s_{i_1})}\\
&\quad + \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \log_2 \frac{1}{P(s_{i_2})}\\
&\quad + \cdots\\
&\quad + \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \log_2 \frac{1}{P(s_{i_n})} && (10)
\end{aligned}$$

式 2-17

$$\begin{aligned}
&\sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \log_2 \frac{1}{P(s_{i_1})} && (11)\\
&= \sum_{(s_{i_1}, s_{i_2}, \ldots, s_{i_n}) \in S^n} P(s_{i_1}) P(s_{i_2}) \cdots P(s_{i_n}) \log_2 \frac{1}{P(s_{i_1})} && (12)
\end{aligned}$$

$$= \sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q} P(s_{i_1})P(s_{i_2}) \cdots P(s_{i_n}) \log_2 \frac{1}{P(s_{i_1})} \tag{13}$$

$$= \sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_{n-1}=1}^{q} P(s_{i_1}) \log_2 \frac{1}{P(s_{i_1})} P(s_{i_2}) \cdots P(s_{i_{n-1}}) \sum_{i_n=1}^{q} P(s_{i_n}) \tag{14}$$

$$= \sum_{i_1=1}^{q} P(s_{i_1}) \log_2 \frac{1}{P(s_{i_1})} \sum_{i_2=1}^{q} P(s_{i_2}) \sum_{i_3=1}^{q} P(s_{i_3}) \cdots \sum_{i_n=1}^{q} P(s_{i_n}) \tag{15}$$

$$= \sum_{i_1=1}^{q} P(s_{i_1}) \log_2 \frac{1}{P(s_{i_1})} \tag{16}$$

$$= \sum_{s_{i_1} \in S} P(s_{i_1}) \log_2 \frac{1}{P(s_{i_1})} \tag{17}$$

$$= H(S) \tag{18}$$

2.5 マルコフ情報源

式 2-21 訂正

$$P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) = P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \tag{19}$$

式 2-23

$$H(S / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) = \sum_{s_i \in S} P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) I(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \tag{20}$$

式 2-24a

$$H(S) = \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \in S^m} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) H(S / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \tag{21}$$

式 2-24b

$$H(S) \tag{22}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \in S^m} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \sum_{s_i \in S} P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \times I(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \tag{23}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \in S^m} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \sum_{s_i \in S} P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \times \log_2 \frac{1}{P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m})} \tag{24}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \in S^m} \sum_{s_i \in S} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \times \log_2 \frac{1}{P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m})} \tag{25}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \in S^{m+1}} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}) P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m}) \times \log_2 \frac{1}{P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m})} \tag{26}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \in S^{m+1}} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \log_2 \frac{1}{P(s_i / s_{j_1}, s_{j_2}, \ldots, s_{j_m})} \tag{27}$$

S が無記憶情報源のとき 式 (2-24b) と式 (2-5a) が一致することの説明

式 2-24b

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \in S^{m+1}} P(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \log_2 \frac{1}{P(s_i)} \tag{28}$$

$$= \sum_{(s_{j_1}, s_{j_2}, \ldots, s_{j_m}, s_i) \in S^{m+1}} P(s_{j_1}) P(s_{j_2}) \cdots P(s_{j_m}) P(s_i) \log_2 \frac{1}{P(s_i)} \tag{29}$$

$$= \sum_{i=1}^{q} \sum_{j_1=1}^{q} \sum_{j_2=1}^{q} \cdots \sum_{j_m=1}^{q} P(s_i) \log_2 \frac{1}{P(s_i)} P(s_{j_1}) P(s_{j_2}) \cdots P(s_{j_m}) \tag{30}$$

$$= \sum_{i=1}^{q} P(s_i) \log_2 \frac{1}{P(s_i)} \sum_{j_1=1}^{q} P(s_{j_1}) \sum_{j_2=1}^{q} P(s_{j_2}) \cdots \sum_{j_m=1}^{q} P(s_{j_m}) \tag{31}$$

$$= \sum_{i=1}^{q} P(s_i) \log_2 \frac{1}{P(s_i)} \cdot 1 \tag{32}$$

$$= \sum_{s_i \in S} P(s_i) \log_2 \frac{1}{P(s_i)} \tag{33}$$

$$= \text{式 2-5a} \tag{34}$$