2.6 随伴情報源

式 2-26

$$\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{P(s_j)P(s_i)}{P(s_j,s_i)}$$

式 2-28

$$\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{P(s_i)}{P(s_i/s_j)} \le 0 \tag{1}$$

$$\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\left(\frac{1}{P(s_i/s_j)}/\frac{1}{P(s_i)}\right) \le 0 \tag{2}$$

$$\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\left(\log\frac{1}{P(s_i/s_j)} - \log\frac{1}{P(s_i)}\right) \le 0 \tag{3}$$

$$\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{1}{P(s_i/s_j)} - \sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{1}{P(s_i)} \le 0 \tag{4}$$

$$\begin{aligned}\sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{1}{P(s_i/s_j)} &\le \sum_{(s_j,s_i)\in S^2} P(s_j,s_i)\log\frac{1}{P(s_i)} \\ &= \sum_{i=1}^{q}\sum_{j=1}^{q} P(s_j,s_i)\log\frac{1}{P(s_i)} \\ &= \sum_{i=1}^{q}\log\frac{1}{P(s_i)}\sum_{j=1}^{q} P(s_j,s_i) \\ &= \sum_{s_i\in S} P(s_i)\log\frac{1}{P(s_i)} \end{aligned}\tag{5}$$

2.7 マルコフ情報源の拡大

式 2-32 の右辺は，状態を決める直前の m 個のシンボルが順に $(s_{j_1}, s_{j_2}, , \ldots s_{j_m})$ であったという状態からスタートして s_{i_1} が出る確率，次に s_{i_2} が出る確率，…，s_{i_n} が出る確率を順に乗じて得る．
式 2-33a は式 2-24b から得る．

$$H(S^n) = \sum_{\sigma_i\in S^n}\sum_{\sigma_j\in S^n} P(\sigma_j,\sigma_i)\log\frac{1}{P(\sigma_i/\sigma_j)}$$

式 2-33b

$$H(S^n) = \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(\sigma_i/\sigma_j)}$$

式 2-34 $\sigma_i = (s_{i_1}, s_{i_2}, \ldots, s_{i_n})$, $\sigma_j = (s_{j_1}, s_{j_2}, \ldots, s_{j_n})$ とすると

$$\begin{aligned} P(\sigma_i/\sigma_j) &= P(s_{i_1}, s_{i_2}, \ldots, s_{i_n}/s_{j_n}) \qquad (6) \\ &= P(s_{i_1}/s_{j_n})P(s_{i_2}/s_{i_1})\cdots P(s_{i_n}/s_{i_{n-1}}) \qquad (7) \end{aligned}$$

式 2-35

$$H(S^n) = \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(\sigma_i/\sigma_j)} \tag{8}$$

$$= \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_1}/s_{j_n})P(s_{i_2}/s_{i_1})\cdots P(s_{i_n}/s_{i_{n-1}})} \tag{9}$$

$$= \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\left\{\log\frac{1}{P(s_{i_1}/s_{j_n})}+\log\frac{1}{P(s_{i_2}/s_{i_1})}+\cdots + \log\frac{1}{P(s_{i_n}/s_{i_{n-1}})}\right\} \tag{10}$$

$$= \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_1}/s_{j_n})}+\sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_2}/s_{i_1})}+\cdots + \sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_n}/s_{i_{n-1}})} \tag{11}$$

式 2-36

$$\sum_{(\sigma_j,\sigma_i)\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_1}/s_{j_n})} \tag{12}$$

$$= \sum_{(s_{j_1},s_{j_2},\ldots,s_{j_n},s_{i_1},s_{i_2},\ldots,s_{i_n})\in S^{2n}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_1}/s_{j_n})} \tag{13}$$

$$= \sum_{(s_{j_n},s_{i_1})\in S^2}\ \sum_{(s_{j_1},s_{j_2},\ldots,s_{j_{n-1}},s_{i_2},s_{i_3},\ldots,s_{i_n})\in S^{2n-2}} P(\sigma_j,\sigma_i)\log\frac{1}{P(s_{i_1}/s_{j_n})} \tag{14}$$

$$= \sum_{(s_{j_n},s_{i_1})\in S^2}\log\frac{1}{P(s_{i_1}/s_{j_n})}\sum_{(s_{j_1},s_{j_2},\ldots,s_{j_{n-1}},s_{i_2},s_{i_3},\ldots,s_{i_n})\in S^{2n-2}} P(s_{j_1},s_{j_2},\ldots,s_{j_n},s_{i_1},s_{i_2},\ldots,s_{i_n})$$

ここで,

$$\sum_{(s_{j_1},s_{j_2},\ldots,s_{j_{n-1}},s_{i_2},s_{i_3},\ldots,s_{i_n})\in S^{2n-2}} P(s_{j_1},s_{j_2},\ldots,s_{j_n},s_{i_1},s_{i_2},\ldots,s_{i_n})$$

は s_{j_n}, s_{i_1} 以外すべての和であるから $P(s_{j_n}, s_{i_1})$ となり式 2-24b とから式 2-36 を得る.

式 2-38 の一行目は記憶のない情報源の情報量を求める式 2-5a から得られる.

式 2-39 は s_{i_1} が出る確率, その後 s_{i_2} が出る確率…と順に乗じたもの.

式 2-40　　　　　(式 2-41 左辺は $H(\overline{S^n})$ の誤り)

$$
\begin{aligned}
H(\overline{S^n}) &= \sum_{(s_{i_1},s_{i_2},\ldots,s_{i_n})\in S^n} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_1})} \\
&\quad + \sum_{(s_{i_1},s_{i_2},\ldots,s_{i_n})\in S^n} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_2}/s_{i_1})} \\
&\qquad \vdots \\
&\quad + \sum_{(s_{i_1},s_{i_2},\ldots,s_{i_n})\in S^n} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_n}/s_{i_{n-1}})} \qquad (15) \\
&= \sum_{s_{i_1}\in S}\sum_{(s_{i_2},s_{i_3},\ldots,s_{i_n})\in S^{n-1}} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_1})} \\
&\quad + \sum_{(s_{i_1},s_{i_2})\in S^2}\sum_{(s_{i_3},s_{i_4},\ldots,s_{i_n})\in S^{n-2}} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_2}/s_{i_1})} \\
&\qquad \vdots \\
&\quad + \sum_{(s_{i_{n-1}},s_{i_n})\in S^2}\sum_{(s_{i_1},s_{i_2},\ldots,s_{i_{n-2}})\in S^{n-2}} P(s_{i_1},s_{i_2},\ldots,s_{i_n})\log\frac{1}{P(s_{i_n}/s_{i_{n-1}})} \qquad (16) \\
&= \sum_{s_{i_1}\in S}\log\frac{1}{P(s_{i_1})}\sum_{(s_{i_2},s_{i_3},\ldots,s_{i_n})\in S^{n-1}} P(s_{i_1},s_{i_2},\ldots,s_{i_n}) \\
&\quad + \sum_{(s_{i_1},s_{i_2})\in S^2}\log\frac{1}{P(s_{i_2}/s_{i_1})}\sum_{(s_{i_3},s_{i_4},\ldots,s_{i_n})\in S^{n-2}} P(s_{i_1},s_{i_2},\ldots,s_{i_n}) \\
&\qquad \vdots \\
&\quad + \sum_{(s_{i_{n-1}},s_{i_n})\in S^2}\log\frac{1}{P(s_{i_n}/s_{i_{n-1}})}\sum_{(s_{i_1},s_{i_2},\ldots,s_{i_{n-2}})\in S^{n-2}} P(s_{i_1},s_{i_2},\ldots,s_{i_n}) \qquad (17) \\
&= \sum_{s_{i_1}\in S} P(s_{i_1})\log\frac{1}{P(s_{i_1})} \qquad (= H(\overline{S})) \\
&\quad + \sum_{(s_{i_1},s_{i_2})\in S^2} P(s_{i_1},s_{i_2})\log\frac{1}{P(s_{i_2}/s_{i_1})} \qquad (= H(S)) \\
&\qquad \vdots \\
&\quad + \sum_{(s_{i_{n-1}},s_{i_n})\in S^2} P(s_{i_{n-1}},s_{i_n})\log\frac{1}{P(s_{i_n}/s_{i_{n-1}})} \qquad (= H(S)) \qquad (18)
\end{aligned}
$$